

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

King Abdulaziz University



Review for The Second Exam-Fall 2012

Hamed Al-Sulami

- أنقر على Start لبدء الاختبار.
- يحتوي هذا الاختبار على ستة وثلاثون سؤالاً.
- عند الانتهاء من الاختبار أنقر على End للحصول على النتيجة.
- بالتوفيق إن شاء الله.

Calculus II
Math202

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Version 1.0



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Answer each of the following.

1. $\int \sin^2 x \cos^3 x \, dx =$

$$\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$$

$$\frac{1}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C$$

$$\frac{1}{3} \cos^3 x - \frac{1}{5} \cos^5 x + C$$

$$\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x + C$$

2. $\int \cos(4x) \sin(3x) dx =$

$$\frac{1}{2} \sin x - \frac{1}{14} \sin(7x) + C$$

$$\frac{1}{2} \cos x + \frac{1}{14} \cos(7x) + C$$

$$\frac{1}{2} \cos x - \frac{1}{14} \cos(7x) + C$$

$$-\frac{1}{2} \cos x + \frac{1}{14} \cos(7x) + C$$

3. To evaluate $\int \frac{x^2}{\sqrt{9-x^2}} dx$ using Trigonometric substitution, we let

$$x = 3 \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

$$x = 3 \sin \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}.$$

$$x = 3 \sec \theta, \quad 0 \leq \theta < \frac{\pi}{2} \text{ or } \pi \leq \theta < \frac{3\pi}{2}$$

$$x = \sec \theta \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

4. $\int \frac{x^2}{\sqrt{9-x^2}} dx =$

$$\frac{9}{2} \tan^{-1} \left(\frac{x}{3} \right) + \frac{x}{2} \sqrt{9-x^2} + C$$

$$\frac{9}{2} \tan^{-1} \left(\frac{x}{3} \right) - \frac{x}{2} \sqrt{9-x^2} + C$$

$$\frac{9}{2} \sin^{-1} \left(\frac{x}{3} \right) + \frac{x}{2} \sqrt{9-x^2} + C$$

$$\frac{9}{2} \sin^{-1} \left(\frac{x}{3} \right) - \frac{x}{2} \sqrt{9-x^2} + C$$

5. $\int_{2\sqrt{2}}^4 \frac{dx}{\sqrt{x^2-4}} =$

$$\ln \left| \frac{1+\sqrt{2}}{2+\sqrt{3}} \right|$$

$$\ln \left| \frac{2+\sqrt{3}}{1+\sqrt{2}} \right|$$

$$1 + \sqrt{2}$$

$$2 + \sqrt{3}$$

$$\begin{aligned} 6. \int \frac{dx}{(x^2 + 1)^{\frac{3}{2}}} = \\ \frac{x}{\sqrt{x^2 + 1}} + C \\ \frac{\sqrt{x^2 + 1}}{x} + C \\ \frac{1}{\sqrt{x^2 + 1}} + C \\ \frac{1}{x\sqrt{x^2 + 1}} + C \end{aligned}$$

$$\begin{aligned} 7. \int \frac{dx}{\sqrt{9 + 4x - x^2}} = \\ \frac{1}{5} \sec^{-1} \left(\frac{x-4}{5} \right) + C \\ \cosh^{-1} \left(\frac{x-4}{5} \right) + C \\ \sinh^{-1} \left(\frac{x-4}{5} \right) + C \\ \sin^{-1} \left(\frac{x-4}{5} \right) + C \end{aligned}$$

8. $\int \frac{\sin^3 x}{\sqrt{\cos x}} dx =$

$$\frac{2}{5}\sqrt{\cos^5 x} + 2\sqrt{\cos x} + C$$

$$\frac{2}{5}\sqrt{\cos^5 x} - 2\sqrt{\cos x} + C$$

$$\frac{2}{5}\sqrt{\sin^5 x} + 2\sqrt{\sin x} + C$$

$$\frac{2}{5}\sqrt{\sin^5 x} - 2\sqrt{\sin x} + C$$

9. $\int \frac{\tan^3 x}{\sqrt{\sec x}} dx =$

$$\frac{2}{3}\sqrt{\tan^3 x} + 2\sqrt{\tan x} + C$$

$$\frac{2}{3}\sqrt{\sec^3 x} - 2\sqrt{\sec x} + C$$

$$\frac{2}{3}\sqrt{\sec^3 x} + \frac{2}{\sqrt{\sec x}} + C$$

$$\frac{2}{3}\sqrt{\sec^3 x} - 2\sqrt{\sec x} + C$$

10. $\int \sinh^3 x \cosh^8 x \, dx =$

$$\frac{1}{11} \cosh^{11} x - \frac{1}{9} \cosh^9 x + C$$

$$\frac{1}{11} \sinh^{11} x + \frac{1}{9} \sinh^9 x + C$$

$$\frac{1}{11} \cosh^{11} x + \frac{1}{9} \cosh^9 x + C$$

$$\frac{1}{11} \sinh^{11} x - \frac{1}{9} \sinh^9 x + C$$

11. $\int \csc^4 x \cot^4 x \, dx =$

$$\frac{1}{7} \cot^7 x + \frac{1}{5} \cot^5 x + C$$

$$\frac{-1}{7} \csc^7 x - \frac{1}{5} \csc^5 x + C$$

$$\frac{1}{7} \csc^7 x + \frac{1}{5} \csc^5 x + C$$

$$\frac{-1}{7} \cot^7 x - \frac{1}{5} \cot^5 x + C$$

12. $\int \tan^2 x \sec x \, dx =$

$$\frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C$$

$$\frac{1}{2} \sec x \tan x - \frac{1}{2} \ln |\sec x + \tan x| + C$$

$$-\frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x|$$

$$\frac{1}{2} \sec x \tan x - \ln |\sec x + \tan x|$$

13. The correct trigonometric substitution for the integral

$$\int \frac{dx}{x^2 \sqrt{4 - 9x^2}} \, dx \text{ is}$$

$$x = \frac{2}{3} \tan \theta$$

$$x = \frac{2}{3} \sec \theta$$

$$x = \frac{2}{3} \sin \theta$$

$$x = \frac{3}{2} \sec \theta$$

14. $\int \frac{x^3}{\sqrt{4+x^2}} dx =$

$$\frac{x^2}{\sqrt{4+x^2}} + \sqrt{4+x^2} + C$$

$$\frac{1}{3}(\sqrt{4+x^2})^3 - 4\sqrt{4+x^2} + C$$

$$x \tan^{-1} x + \frac{1}{\sqrt{4+x^2}} + C$$

$$x \tan^{-1} x + C$$

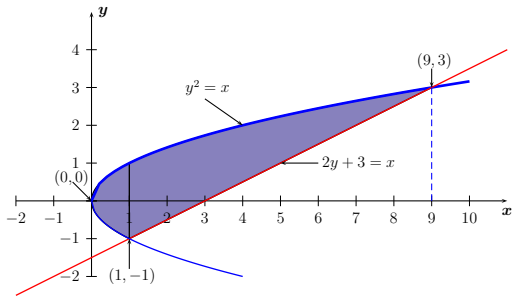
15. The integral that represent the area of the region enclosed by $y^2 = x$ and $x - 2y = 3$ is

$$\int_1^9 \left[\sqrt{x} - \left(\frac{x-3}{2} \right) \right] dx.$$

$$\int_{-1}^3 [2y + 3 - y^2] dy.$$

$$\int_0^1 2\sqrt{x} dx.$$

$$\int_0^3 [2y + 3 - y^2] dy.$$



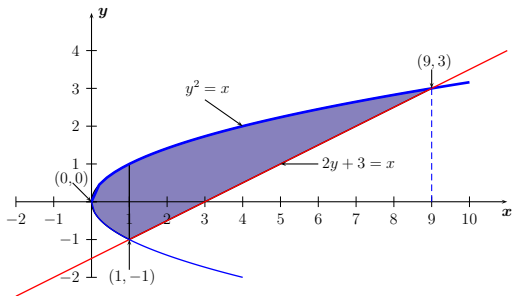
16. The area of the region enclosed by $y^2 = x$ and $x - 2y = 3$ is

$$\frac{32}{3}$$

$$\frac{31}{3}$$

$$32$$

$$\frac{31}{2}$$



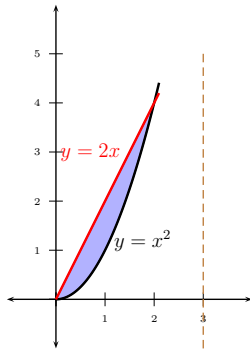
17. The integral that represents the volume of the solid obtained by revolving the region **R**, $y = x^2$, $y = 2x$ (see the graph to the right) about $x = 3$ is

$$\int_0^2 2\pi(3-x)(2x-x^2) dx.$$

$$\int_0^2 \pi(3-x)(2x-x^2) dx.$$

$$\int_0^4 2\pi[(3-\frac{y}{2})^2 - (3-\sqrt{y})^2] dy.$$

$$\int_0^4 \pi[(3-\frac{y}{2})^2 - (3-\sqrt{y})^2] dy.$$



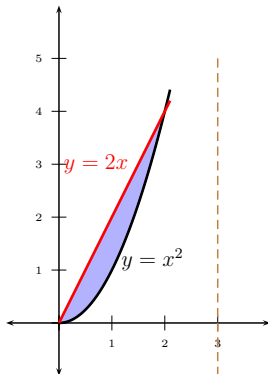
18. The volume of the solid obtained by revolving the region $\mathbf{R}$, $y = x^2$, $y = 2x$ (see the graph to the right) about $x = 3$ is

$$\frac{32\pi}{3}.$$

$$\frac{8\pi}{3}.$$

$$\frac{16}{3}.$$

$$\frac{16\pi}{3}.$$



19. To evaluate $\int \frac{\sqrt{x^2-16}}{x^4} dx$ using Trigonometric substitution, we let

$$x = 4 \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

$$x = 4 \sin \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}.$$

$$x = 4 \sec \theta, \quad 0 \leq \theta < \frac{\pi}{2} \text{ or } \pi \leq \theta < \frac{3\pi}{2}$$

$$x = 4 \sec \theta \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}.$$

20. $\int \frac{\sqrt{x^2-16}}{x^4} dx =$

$$\frac{(x^2-16)^{3/2}}{48x^3} + C$$

$$\frac{(x^2-16)^{1/2}}{48x^3} + C$$

$$\frac{(x^2-16)^{3/2}}{24x^3} + C$$

$$\frac{(x^2-16)^{1/2}}{24x^3} + C$$

21. $\int \tan^3 x \sec^3 x \, dx =$

$$\frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$$

$$\frac{1}{5} \tan^5 x - \frac{1}{3} \tan^3 x + C$$

$$\frac{1}{5} \sec^5 x + \frac{1}{3} \sec^3 x + C$$

$$\frac{1}{5} \tan^5 x + \frac{1}{3} \tan^3 x + C$$

22. The form of the partial fraction decomposition of $\frac{x^2 + 3x + 6}{x^3 - 4x} =$

$$\frac{A}{x} + \frac{Bx + C}{x^2 - 4}$$

$$\frac{A}{x^2} + \frac{B}{x - 4}$$

$$\frac{A}{x} + \frac{B}{x - 2} + \frac{C}{(x - 2)^2}$$

$$\frac{A}{x} + \frac{B}{x - 2} + \frac{C}{x + 2}$$

23. The form of the partial fraction decomposition of the

rational function $\frac{x^2 + 1}{(x - 1)(x^2 + 2x + 2)^2}$ is

$$\frac{Ax + B}{x - 1} + \frac{Cx + D}{x^2 + 2x + 2} + \frac{Ex + F}{(x^2 + 2x + 2)^2}$$

$$\frac{A}{x - 1} + \frac{Bx + C}{x^2 + 2x + 2} + \frac{Dx + E}{(x^2 + 2x + 2)^2}$$

$$\frac{A}{x - 1} + \frac{Bx + C}{(x^2 + 2x + 2)^2}$$

$$\frac{A}{x - 1} + \frac{B}{x^2 + 2x + 2} + \frac{Cx + D}{(x^2 + 2x + 2)^2}$$

24. By using long division, we have $\frac{x^4-2x^2+4x+1}{x^3-x^2-x+1} =$

$$x + 1 + \frac{4x}{x^3-x^2-x+1}$$

$$x - 1 + \frac{4x}{x^3-x^2-x+1}$$

$$x + \frac{4x}{x^3-x^2-x+1}$$

$$-x + 1 + \frac{4x}{x^3-x^2-x+1}$$

25. The form of the partial fraction decomposition of the remainder is $\frac{4x}{x^3-x^2-x+1} = \frac{4x}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$. Solving for A, B , and C , we get

$$A = 1, B = 2, C = -1$$

$$A = -1, B = 2, C = -1$$

$$A = 1, B = -2, C = -1$$

$$A = 1, B = 2, C = 1$$

26. $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1} dx =$

$$\frac{x^2}{2} - x + \ln|x-1| - \frac{2}{x-1} - \ln|x+1| + C$$

$$\frac{x^2}{2} + x - \ln|x-1| - \frac{2}{x-1} + \ln|x+1| + C$$

$$-\frac{x^2}{2} + x + \ln|x-1| + \frac{2}{x-1} + \ln|x+1| + C$$

$$\frac{x^2}{2} + x + \ln|x-1| - \frac{2}{x-1} - \ln|x+1| + C$$

$$27. \int \frac{3^x \ln 3}{3^{2x} + 1} dx =$$

$$\sin^{-1}(3^x) + C$$

$$\tan^{-1}(3^x) + C$$

$$\ln(1 + 3^{2x}) + C$$

$$\frac{(1+3^{2x})^2}{2} + C$$

$$28. \int \frac{1}{\sqrt{10x-x^2}} dx =$$

$$\sin^{-1}(x-5) + C$$

$$\tan^{-1}(x-5) + C$$

$$\sin^{-1}\left(\frac{x-5}{5}\right) + C$$

$$\sqrt{10x} + \sin^{-1} x + C$$

29. $\int_0^6 \frac{1}{x-1} dx =$

$\ln 5$

$\ln 2$

Divergent

$\ln 3$

30. $\int_e^\infty \frac{1}{x(\ln x)^3} dx =$

1

e

Divergent

$\frac{1}{2}$

Answers:

Points:

Percent:

Letter Grade:

Solutions to Quizzes

Solution to 1.

$$\begin{aligned}\int \sin^2 x \cos^3 x \, dx &= \int \sin^2 x \cos^2 x \cos x \, dx && \text{use } \cos^2 x = 1 - \sin^2 x \\&= \int \sin^2 x (1 - \sin^2 x) \cos x \, dx && \text{Let } u = \sin x, \, du = \cos x \, dx \\&= \int u^2 (1 - u^2) \, du \\&= \int u^2 - u^4 \, du \\&= \frac{u^3}{3} - \frac{u^5}{5} + C && \text{replace } u = \sin x. \\&= \frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C.\end{aligned}$$



Solution to 2.

$$\begin{aligned}& \int \cos(4x) \sin(3x) dx \\&= \int \frac{\sin(3x - 4x) + \sin(3x + 4x)}{2} dx \quad \text{use } \sin A \cos B = \frac{\sin(A - B) + \sin(A + B)}{2}. \\&= \frac{1}{2} \int [\sin(-x) + \sin(7x)] dx \quad \text{use } \sin(-x) = -\sin x. \\&= \int [-\sin x + \sin(7x)] dx \quad \text{use } \int \sin(ax) dx = \frac{-1}{a} \cos x + C. \\&= \frac{1}{2} \cos x - \frac{1}{14} \cos(7x) + C\end{aligned}$$



Solution to 3. Since we have $\sqrt{9 - x^2}$ then $x = 3 \sin \theta$, $\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}$. 

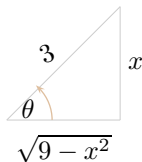
Solution to 4.

Since we have $\sqrt{9-x^2}$
then

$$x = 3 \sin \theta$$

$$dx = 3 \cos \theta d\theta$$

$$x^2 = (3 \sin \theta)^2 = 9 \sin^2 \theta \sqrt{9-x^2} = 3 \cos \theta.$$



$$\begin{aligned}\int \frac{x^2}{\sqrt{9-x^2}} dx &= \int \frac{9 \sin^2 \theta}{\cancel{3 \cos \theta}} \cancel{3 \cos \theta} d\theta \\&= 9 \int \sin^2 \theta d\theta && \text{use } \sin^2 A = \frac{1 - \cos(2A)}{2} . \\&= 9 \int \frac{1 - \cos(2\theta)}{2} d\theta \\&= \frac{9}{2} [\theta - \frac{1}{2} \sin(2\theta)] + C \\&= \frac{9}{2} [\theta - \sin \theta \cos \theta] + C \\&= \frac{9}{2} \left[\sin^{-1} \left(\frac{x}{3} \right) - \frac{x}{3} \frac{\sqrt{9-x^2}}{3} \right] + C \\&= \frac{9}{2} \sin^{-1} \left(\frac{x}{3} \right) - \frac{x}{2} \sqrt{9-x^2} + C.\end{aligned}$$



Solution to 5.

$$\int_{2\sqrt{2}}^4 \frac{dx}{\sqrt{x^2 - 4}} = \int_{2\sqrt{2}}^4 \frac{1}{\sqrt{x^2 - 2^2}} dx$$

$$\text{use } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln |x + \sqrt{x^2 - a^2}| + C.$$

$$= \left[\ln |x + \sqrt{x^2 - 4}| \right]_{2\sqrt{2}}^4$$

$$= \left[\ln |4 + \sqrt{12}| - \ln |2\sqrt{2} + 2| \right]$$

$$= \ln \left| \frac{4 + 2\sqrt{3}}{2 + 2\sqrt{2}} \right|$$

$$= \ln \left| \frac{2(2 + \sqrt{3})}{2(1 + \sqrt{2})} \right|$$

$$= \ln \left| \frac{2 + \sqrt{3}}{1 + \sqrt{2}} \right|.$$

Another solution: Let $x = 2 \sec \theta$,
 $dx = 2 \sec \theta \tan \theta d\theta$
 $\sqrt{x^2 - 4} = 2 \tan \theta$.

Now,

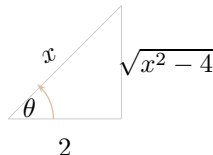
$$\theta = \sec^{-1} \left(\frac{x}{2} \right)$$

hence when $x = 2\sqrt{2}$ then

$$\theta = \sec^{-1} (\sqrt{2}) = \frac{\pi}{4}.$$

Also when $x = 4$ then

$$\theta = \sec^{-1} (2) = \frac{\pi}{3}$$



$$\begin{aligned}\int_{2\sqrt{2}}^4 \frac{dx}{\sqrt{x^2-4}} &= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{2 \tan \theta} 2 \sec \theta \tan \theta d\theta \\&= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sec \theta d\theta \\&= [\ln |\sec \theta + \tan \theta|]_{\frac{\pi}{4}}^{\frac{\pi}{3}} \\&= [\ln |2 + \sqrt{3}| - \ln |\sqrt{2} + 1|] \\&= \ln \left| \frac{2 + \sqrt{3}}{1 + \sqrt{2}} \right|.\end{aligned}$$

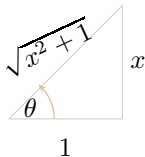


Solution to 6. Let

$$x = \tan \theta,$$

$$dx = \sec^2 \theta d\theta$$

$$\sqrt{x^2 + 1} = \sec \theta.$$



$$\begin{aligned} \int \frac{dx}{(\sqrt{x^2 + 1})^3} &= \int \frac{\sec^2 \theta d\theta}{\sec^3 \theta} \\ &= \int \frac{d\theta}{\sec \theta} \\ &= \int \cos \theta d\theta \\ &= \int \cos \theta d\theta \\ &= \sin \theta + C = \frac{x}{\sqrt{x^2 + 1}} + C. \end{aligned}$$



Solution to 7.

$$\begin{aligned}9 + 8x - x^2 &= -x^2 + 8x + 9, \\&= -(x^2 - 8x + 4^2 - 4^2) + 9, \\&= -(x^2 - 8x + 4^2) + 16 + 9, \\&= 25 - (x - 4)^2.\end{aligned}$$

$$\begin{aligned}\int \frac{dx}{\sqrt{9 + 4x - x^2}} &= \int \frac{1}{\sqrt{5^2 - (x - 4)^2}} dx \\&= \sin^{-1} \left(\frac{x - 4}{5} \right) + C.\end{aligned}$$



Solution to 8.

$$\begin{aligned}\int \frac{\sin^3 x}{\sqrt{\cos x}} dx &= \int \frac{\sin^2 x}{\sqrt{\cos x}} \sin x dx \\&= \int \frac{1 - \cos^2 x}{\sqrt{\cos x}} \sin x dx \\&= - \int \frac{1 - \cos^2 x}{\sqrt{\cos x}} (-\sin x) dx \\&= \int \frac{\cos^2 x - 1}{\sqrt{\cos x}} d(\cos x) \\&= \int (\cos^{\frac{3}{2}} x - \cos^{\frac{-1}{2}} x) d(\cos x) \\&= \frac{2}{5} \cos^{\frac{5}{2}} x + 2 \cos^{\frac{1}{2}} x + C \\&= \frac{2}{5} \sqrt{\cos^5 x} + 2\sqrt{\cos x} + C\end{aligned}$$



Solution to 9.

$$\begin{aligned}\int \frac{\tan^3 x}{\sqrt{\sec x}} dx &= \int \frac{\tan^2 x}{\sec x \sqrt{\sec x}} \sec x \tan x dx \\&= \int \frac{\sec^2 x - 1}{\sec^{\frac{3}{2}} x} \sec x \tan x dx \\&= \int (\sec^{\frac{1}{2}} x - \sec^{\frac{-3}{2}} x) d(\sec x) \\&= \frac{2}{3} \sec^{\frac{3}{2}} x + 2 \sec^{\frac{-1}{2}} x + C \\&= \frac{2}{3} \sqrt{\sec^3 x} + \frac{2}{\sqrt{\sec x}} + C\end{aligned}$$



Solution to 10.

$$\begin{aligned}\int \sinh^3 x \cosh^8 x \, dx &= \int \sinh^2 x \cosh^8 x \sinh x \, dx \\&= \int (\cosh^2 x - 1) \cosh^8 x \sinh x \, dx \\&= \int (\cosh^{10} x - \cosh^8 x) d(\cosh x) \\&= \frac{1}{11} \cosh^{11} x - \frac{1}{9} \cosh^9 x + C.\end{aligned}$$



Solution to 11.

$$\begin{aligned}\int \csc^4 x \cot^4 x \, dx &= \int \csc^2 x \cot^4 x \csc^2 x \, dx \\&= - \int (1 + \cot^2 x) \cot^4 x (-\csc^2 x) \, dx \\&= - \int (\cot^4 x + \cot^6 x) d(\cot x) \\&= \frac{-1}{7} \cot^7 x - \frac{1}{5} \cot^5 x + C.\end{aligned}$$



Solution to 12.

$$\begin{aligned}\int \tan^2 x \sec x \, dx &= \int (\sec^2 x - 1) \sec x \, dx \\&= \int \sec^3 x \, dx - \int \sec x \, dx \\&= \frac{1}{2} \sec x \tan x + \frac{1}{2} \int \sec x \, dx - \int \sec x \, dx \\&= \frac{1}{2} \sec x \tan x - \frac{1}{2} \int \sec x \, dx \\&= \frac{1}{2} \sec x \tan x - \frac{1}{2} \ln |\sec x + \tan x| + C.\end{aligned}$$



Solution to 13. $\int \frac{dx}{x^2 \sqrt{4 - 9x^2}} dx = \int \frac{dx}{x^2 \sqrt{2^2 - (3x)^2}} dx$

Hence $3x = 2 \sin \theta$. Therefore $x = \frac{2}{3} \sin \theta$. ■

Solution to 14.

$$w = \sqrt{4 + x^2}$$

$$w^2 = 4 + x^2$$

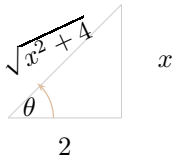
$$x^2 = w^2 - 4$$

$$2x \, dx = 2w \, dw$$

$$x \, dx = w \, dw$$

$$\begin{aligned}\int \frac{x^3}{\sqrt{x^2 + 4}} \, dx &= \int \frac{x^2}{\sqrt{x^2 + 4}} x \, dx \\&= \int \frac{w^2 - 4}{w} w \, dw \\&= \int w^2 - 4 \, dw \\&= \frac{1}{3}w^3 - 4w + C \\&= \frac{1}{3}(\sqrt{x^2 + 4})^3 - 4\sqrt{x^2 + 4} + C \\&= \frac{1}{3}(x^2 + 4)^{3/2} - 4\sqrt{x^2 + 4} + C.\end{aligned}$$

Another solution: Let $x = 2 \tan \theta$,
 $dx = 2 \sec^2 \theta d\theta$
 $x^3 = 8 \tan^3 \theta$
 $\sqrt{x^2 + 4} = 2 \sec \theta.$



$$\begin{aligned}\int \frac{x^3}{\sqrt{x^2+4}} dx &= \int \frac{8 \tan^3 \theta}{2 \sec \theta} 2 \sec^2 \theta d\theta \\&= 8 \int \tan^3 \theta \sec \theta d\theta \\&= 8 \int \tan^2 \theta \sec \theta \tan \theta d\theta && \text{use } 1 + \tan^2 A = \sec^2 A. \\&= 8 \int (\sec^2 \theta - 1) \sec \theta \tan \theta d\theta && \text{use } u = \sec \theta \Rightarrow du = \sec \theta \tan \theta d\theta. \\&= 8 \int (u^2 - 1) du \\&= \frac{8}{3} u^3 - 8u + C \\&= \frac{8}{3} \sec^3 \theta - 8 \sec \theta + C \\&= \frac{1}{3} \left(\frac{\sqrt{x^2+4}}{2} \right)^3 - 8 \frac{\sqrt{x^2+4}}{2} + C \\&= \frac{1}{3} (x^2+4)^{3/2} - 4\sqrt{x^2+4} + C.\end{aligned}$$



Solution to 15. We find the points of intersection by solving $y^2 = 2y+3$. Hence $y^2-2y-3 = 0$, thus $(y-3)(y+1) = 0$. Therefore $y = 3$ or $y = -1$. It is easier to integrate with respect to y . From the figure we can see that $x_{\text{right}} = 2y + 3$ and $x_{\text{left}} = y^2$. Hence $A = \int_{-1}^3 [2y + 3 - y^2] dy$. ■

Solution to 16. We find the points of intersection by solving $y^2 = 2y+3$. Hence $y^2-2y-3 = 0$, thus $(y-3)(y+1) = 0$. Therefore $y = 3$ or $y = -1$. It is easier to integrate with respect to y . From the figure we can see that $x_{\text{right}} = 2y + 3$ and $x_{\text{left}} = y^2$. Hence $A = \int_{-1}^3 [2y + 3 - y^2] dy$.

$$\begin{aligned} A &= \int_{-1}^3 [2y + 3 - y^2] dy \\ &= \left[y^2 + 3y - \frac{y^3}{3} \right]_{-1}^3 \\ &= \left[(9 + 9 - 9) - (1 - 3 + \frac{1}{3}) \right] \\ &= \left[9 + 2 - \frac{1}{3} \right] = \frac{32}{3} \end{aligned}$$



Solution to 17. If we draw a thin rectangle through the region that is perpendicular to the axis of revolution $x = 3$, we see it intersects the y -axis. Hence we should integrate with respect to y . We find the points of intersection by solving $x^2 = 2x \Leftrightarrow x^2 - 2x = 0 \Leftrightarrow x(x-2) = 0$ Thus $x = 0$ and $x = 2$ Hence $y = 0$ and $y = 4$. Thus the intersection points are $(0,0)$ and $(2,4)$. write the equations of the graph as variable of y , $y = x^2 \Rightarrow x = \sqrt{y}$ and $y = 2x \Rightarrow x = \frac{y}{2}$. The cross-section at any $y \in [0, 4]$ perpendicular to the $x = 3$ is an annular "washer-shaped" with inner radius equal to the distance from the $x = 3$ to the nearest graph plus the distance from the y -axis to the axis of revolution $x = -1$, and hence $r_{\text{in}} = 3 - x_{\text{right}} = 3 - \sqrt{y}$. The outer radius equal to the distance from the $x = 3$ to the farthest graph $x = \frac{y}{2}$ and hence $r_{\text{out}} = 3 - x_{\text{left}} = 3 - \frac{y}{2}$. Then

$$\begin{aligned} V &= \int_c^d \pi[(\text{outer radius})^2 - (\text{inner radius})^2] dy \\ &= \int_0^4 \pi[(3 - \frac{y}{2})^2 - (3 - \sqrt{y})^2] dy \end{aligned}$$



Solution to 18. If we draw a thin rectangle through the region that is perpendicular to the axis of revolution $x = 3$, we see it intersects the y -axis. Hence we should integrate with respect to y . We find the points of intersection by solving $x^2 = 2x \Leftrightarrow x^2 - 2x = 0 \Leftrightarrow x(x-2) = 0$ Thus $x = 0$ and $x = 2$ Hence $y = 0$ and $y = 4$. Thus the intersection points are $(0, 0)$ and $(2, 4)$. write the equations of the graph as variable of y , $y = x^2 \Rightarrow x = \sqrt{y}$ and $y = 2x \Rightarrow x = \frac{y}{2}$. The cross-section at any $y \in [0, 4]$ perpendicular to the $x = 3$ is an annular "washer-shaped" with inner radius equal to the distance from the $x = 3$ to the nearest graph plus the distance from the y -axis to the axis of revolution $x = -1$, and hence $r_{\text{in}} = 3 - x_{\text{right}} = 3 - \sqrt{y}$. The outer radius equal to the distance from the $x = 3$ to the farthest graph $x = \frac{y}{2}$

and hence $r_{\text{out}} = 3 - x_{\text{left}} = 3 - \frac{y}{2}$. Then

$$\begin{aligned} V &= \int_c^d \pi[(\text{outer radius})^2 - (\text{inner radius})^2] dy \\ &= \int_0^4 \pi[(3 - \frac{y}{2})^2 - (3 - \sqrt{y})^2] dy \\ &= \pi \int_0^4 [9 - 3y + \frac{y^2}{4} - 9 + 6\sqrt{y} - y] dy \\ &= \pi \int_0^4 [\frac{y^2}{4} - 4y + 6y^{1/2}] dy \\ &= \pi [\frac{y^3}{12} - 2y^2 + 4y^{3/2}]_0^4 \\ &= \frac{16\pi}{3} \end{aligned}$$



Solution to 19. Since we have $\sqrt{x^2 - 16}$ then $x = 4 \sec \theta$, $0 \leq \theta < \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$. ■

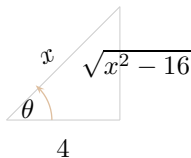
Solution to 20.

Since we have $\sqrt{x^2 - 16}$
then

$$x = 4 \sec \theta$$

$$dx = 4 \sec \theta \tan \theta d\theta$$

$$x^4 = (4 \sec \theta)^2 = 128 \sec^4 \theta \sqrt{x^2 - 16} = 4 \tan \theta.$$



$$\begin{aligned}\frac{\sqrt{x^2 - 16}}{x^4} &= \int \frac{4 \tan \theta}{128 \sec^4 \theta} 4 \sec \theta \tan \theta d\theta \\&= \frac{1}{16} \int \frac{\tan^2 \theta}{\sec^3 \theta} d\theta && \text{use } \tan A = \frac{\sin A}{\cos A}, \frac{1}{\sec A} = \cos A. \\&= \frac{1}{16} \int \sin^2 \theta \cos \theta d\theta && \text{use } u = \sin \theta, du = \cos \theta. \\&= \frac{1}{16} \int u^2 du \\&= \frac{1}{16} \frac{u^3}{3} + C \\&= \frac{\sin^3 \theta}{48} + C && \text{use } \sin \theta = \frac{\sqrt{x^2 - 16}}{x}. \\&= \frac{\left(\frac{\sqrt{x^2 - 16}}{x}\right)^3}{48} + C \\&= \frac{(x^2 - 16)^{3/2}}{48x^3}.\end{aligned}$$



Solution to 21.

$$\begin{aligned}\int \tan^3 x \sec^3 x \, dx &= \int \tan^2 x \sec^2 x \sec x \tan x \, dx && \text{use } \tan^2 A = \sec^2 A - 1. \\&= \int (\sec^2 x - 1) \sec^2 x \sec x \tan x \, dx && \text{let } u = \sec x, \, du = \sec x \tan x \, dx. \\&= \int (u^2 - 1) u^2 \, du \\&= \int (u^4 - u^2) \, du \\&= \frac{u^5}{5} - \frac{u^3}{3} + C \\&= \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C\end{aligned}$$



Solution to 22. Since $x^3 - 4x = x(x^2 - 4) = x(x - 2)(x + 2)$, then we have three distinct linear factors

$$\begin{aligned}\frac{x^2 + 3x + 6}{x^3 - 4x} &= \frac{x^2 + 3x + 6}{x(x - 2)(x + 2)} \\ &= \frac{A}{x} + \frac{B}{x - 2} + \frac{C}{x + 2}\end{aligned}$$



Solution to 23. Since $(x^2 + 2x + 2)^2$ is irreducible, then we have a linear factor and two repeated irreducible factors

$$\frac{x^2 + 1}{(x - 1)(x^2 + 2x + 2)^2} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 2x + 2} + \frac{Dx + E}{(x^2 + 2x + 2)^2}$$



Solution to 24.

Using long division we have

$$\begin{array}{r}
 \overline{) \begin{array}{l} x^4 - 2x^2 + 4x + 1 \\ -x^4 - +x^3 - +x^2 + -x \\ \hline x^3 - x^2 + 3x + 1 \\ -x^3 - +x^2 - +x + -1 \\ \hline 4x \end{array}} \\
 x^3 - x^2 - x + 1
 \end{array}$$

Hence $\frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} = x + 1 + \frac{4x}{x^3 - x^2 - x + 1}$. ■

Solution to 25. Since $x^3 - x^2 - x + 1 = (x - 1)^2(x + 1)$,
then by Linear Factor Rule

$$\frac{4x}{(x - 1)^2(x + 1)} = \frac{4x}{(x - 1)^2(x + 1)}$$

$$\frac{4x}{(x - 1)^2(x + 1)} = \frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{x + 1}$$

Multiply both sides by

$$(x - 1)^2(x + 1).$$

$$4x = A(x - 1)(x + 1) + B(x + 1) + C(x - 1)^2$$

$$\text{if } x = 1 : 4 = 2B \Rightarrow B = 2$$

$$\text{if } x = -1 : -4 = 4C \Rightarrow C = -1$$

$$\text{if } x = 0 : 0 = -A + B + C \Rightarrow A = B + C = 1.$$

Hence $A = 1, B = 2, C = -1$. 

Solution to 26. Since $\frac{x^4-2x^2+4x+1}{x^3-x^2-x+1} = x + 1 + \frac{4x}{x^3-x^2-x+1}$ and $\frac{4x}{x^3-x^2-x+1} = \frac{1}{x-1} + \frac{2}{(x-1)^2} + \frac{-1}{x+1}$, then $\frac{x^4-2x^2+4x+1}{x^3-x^2-x+1} = x + 1 + \frac{1}{x-1} + \frac{2}{(x-1)^2} + \frac{-1}{x+1}$. Hence

$$\begin{aligned} \int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx &= \\ \int (x + 1) dx + \int \frac{1}{x - 1} dx + \int \frac{2}{(x - 1)^2} dx + \int \frac{-1}{x + 1} dx \\ &= \int (x + 1) dx + \int \frac{1}{x - 1} dx + \int 2(x - 1)^{-2} dx - \int \frac{1}{x + 1} dx \\ &= \frac{x^2}{2} + x + \ln |x - 1| - \frac{2}{x - 1} - \ln |x + 1| + C \\ &= \frac{x^2}{2} + x - \frac{2}{x - 1} + \ln \left| \frac{x - 1}{x + 1} \right| + C. \end{aligned}$$



Solution to 27. Since $(3^x)' = 3^x \ln 3$ and $\int \frac{f'(x)}{1+[f(x)]^2} dx = \tan^{-1}(f(x)) + C$, then $\int \frac{3^x \ln 3}{3^{2x} + 1} dx = \tan^{-1}(3^x) + C$. ■

Solution to 28. By completing the square we have

$$10x - x^2 = -x^2 + 10x$$

dotted

$$= -[(x - 5)^2 - 25]$$

$$= 25 - (x - 5)^2.$$

$$\begin{aligned}\int \frac{1}{\sqrt{10x - x^2}} dx &= \int \frac{1}{\sqrt{25 - (x - 5)^2}} dx \text{ Use } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + C. \\ &= \sin^{-1} \left(\frac{x - 5}{5} \right) + C\end{aligned}$$



Solution to 29. Since $\frac{1}{x-1}$ is discontinuous at $1 \in [0, 6]$

we have improper integral. Hence $\int_0^6 \frac{1}{x-1} dx = \int_0^1 \frac{1}{x-1} dx +$

$$\int_1^6 \frac{1}{x-1} dx.$$

$$\begin{aligned}\int_0^1 \frac{1}{x-1} dx &= \lim_{t \rightarrow 1^-} \int_0^t \frac{1}{x-1} dx \\&= \lim_{t \rightarrow 1^-} [\ln |x-1|]_0^t \\&= \lim_{t \rightarrow 1^-} [\ln |t-1| - \ln 1] \\&= -\infty.\end{aligned}$$

Hence $\int_0^1 \frac{1}{x-1} dx$ is divergent Thus $\int_0^6 \frac{1}{x-1} dx$ is divergent.



Solution to 30.

$$\begin{aligned}\int_e^{\infty} \frac{1}{x(\ln x)^3} dx &= \lim_{t \rightarrow \infty} \int_e^t \frac{1}{x(\ln x)^3} dx \\&= \lim_{t \rightarrow \infty} \int (\ln x)^{-3} \frac{1}{x} dx \text{ Use } \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C. \\&= \lim_{t \rightarrow \infty} \left[\frac{-1}{2(\ln x)^2} \right]_e^t \\&= \lim_{t \rightarrow \infty} \left[\frac{-1}{(\ln t)^2} + \frac{1}{2} \right] \\&= \frac{1}{2}.\end{aligned}$$

Hence $\int_e^{\infty} \frac{1}{x(\ln x)^3} dx = \frac{1}{2}$ converge. ■